Modelling the feeding behaviour of a population of grey mouse lemurs via mixed hidden Markov models

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1. Data description

2. Modelling via HMMs
   - HMM for a single animal
   - HMM for multiple animals

3. Current research

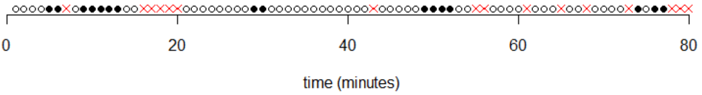
Grey mouse lemur

- largest mouse lemur (up to 70 grams)
- lives in Madagascar, mainly in high trees
- night-active
- food: fruits, leaves, insects, ...

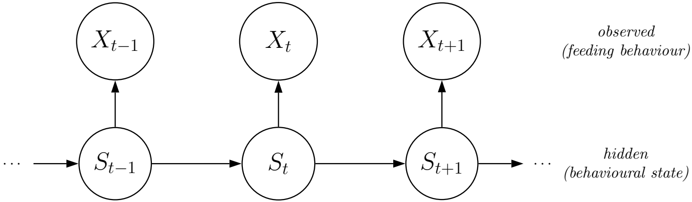
Data

- 16 female & 38 male lemurs
- feeding behaviour observed at 1-minute intervals
  - 0 (not feeding), 1 (feeding), NA (missing/out of sight)
  - time frame: 18 – 23h
  - on at most 7 different days
  - lots of missing data
- available covariates: body mass & gender

Sample:



Basic HMM structure



- X_t: **State-dependent process**
  $\longrightarrow$ 0 (not feeding) or 1 (feeding)

- S_t: **Markov chain**
  $\longrightarrow$ 0/"sated" or 1/"hungry"

$\longrightarrow$ *special case of a state-space model (with finite state space)*

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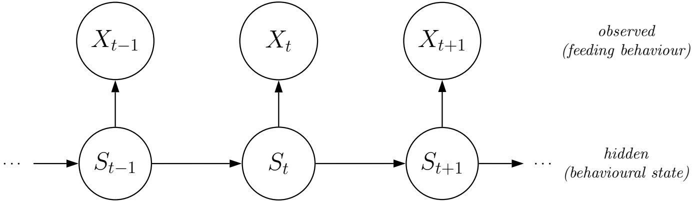

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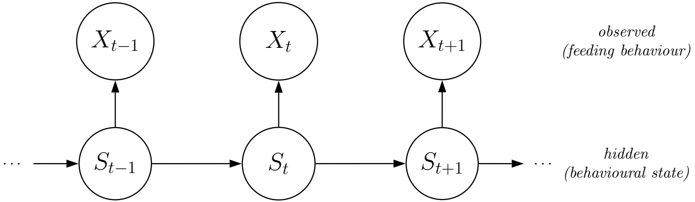

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Model for succession of behavioural states

Transition probabilities are modelled **time-dependently**:

$$\text{logit}(\gamma_{ii}^{(t)}) = \beta_{0,i} + \beta_{1,i} t, \quad i = 1, 2, \quad t = 0, 1, 2, \ldots$$

$$\text{where } \gamma_{ij}^{(t)} = \mathbb{P}\big(S_{t+1} = j \,|\, S_t = i\big)$$

Transition probability matrix (at time t):

$$\Gamma^{(t)} = \begin{pmatrix} \gamma_{11}^{(t)} & \gamma_{12}^{(t)} \\ \gamma_{21}^{(t)} & \gamma_{22}^{(t)} \end{pmatrix}$$

Initial distribution:

$$\delta = \big(\mathbb{P}(S_t = 1), (\mathbb{P}(S_t = 2)\big)$$

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Observation model

State-dependent (Bernoulli) distributions:

$$\mathbb{P}\big(X_t = 1 \,|\, S_t = k\big) = \pi_k$$
$$\mathbb{P}\big(X_t = 0 \,|\, S_t = k\big) = 1 - \pi_k$$

"hungry" state: π_k relatively large (typically ≈ 0.9)
"sated" state: π_k close to 0

Likelihood

$$\mathcal{L} = \boldsymbol{\delta}\mathbf{P}_{\boldsymbol{\pi}}(x_1)\boldsymbol{\Gamma}^{(1)}\mathbf{P}_{\boldsymbol{\pi}}(x_2)\boldsymbol{\Gamma}^{(2)} \cdot \ldots \cdot \mathbf{P}_{\boldsymbol{\pi}}(x_{T-1})\boldsymbol{\Gamma}^{(T-1)}\mathbf{P}_{\boldsymbol{\pi}}(x_T)\mathbf{1}^t$$

where $\mathbf{P}_{\boldsymbol{\pi}}(x_t) = \text{diag}\big(\pi_1^{x_t}(1-\pi_1)^{1-x_t}, \pi_2^{x_t}(1-\pi_2)^{1-x_t}\big)$
(for missing observations: 2×2-identity matrix)

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Strategies for dealing with populations in the HMM framework

- parameter set common to all subjects
  $\longrightarrow$ neglects any possible heterogeneity across subjects
- each of the parameters subject-specific
  $\longrightarrow$ enormous number of parameters
  $\longrightarrow$ impedes the possibility of comparing the resulting models
- some parameters common to all subjects, others subject-spec.
  $\longrightarrow$ special case: random effects
  $\longrightarrow$ computationally demanding
- subject-specific covariates
  $\longrightarrow$ requires that suitable covariates are available

Model for multiple animals

Transition probabilities:

$$\text{logit}(\gamma_{ii}^{(t,m)}) = \beta_{0,i} + \beta_{1,i} t + \beta_{2,i} \text{sex}^{(m)} + \beta_{3,i} \text{mass}^{(m)}$$

State-dependent feeding probabilities:

$$\pi_{i,m} \overset{iid}{\sim} \mathcal{B}eta(a_i, b_i), \qquad m = 1, \ldots, M \, (= 54)$$

Likelihood:

$$\mathcal{L} = \prod_{m=1}^{M} \int_0^1 \int_0^1 \boldsymbol{\delta} \mathbf{P}_{\boldsymbol{\pi}}(x_{1,m}) \boldsymbol{\Gamma}^{(1,m)} \mathbf{P}_{\boldsymbol{\pi}}(x_{2,m}) \boldsymbol{\Gamma}^{(2,m)} \cdot \ldots$$
$$\ldots \cdot \boldsymbol{\Gamma}^{(T-1,m)} \mathbf{P}_{\boldsymbol{\pi}}(x_{T_m,m}) \mathbf{1}^t f_1(\pi_1) f_2(\pi_2) d\pi_1 d\pi_2$$

Estimation

- Monte Carlo expectation–maximization
  $\longrightarrow$ E-step requires simulation methods
  $\longrightarrow$ M-step might require numerical optimization
- Numerical likelihood maximization
  $\longrightarrow$ integrals must be approximated numerically

Both methods are computationally demanding!

Results (part I)

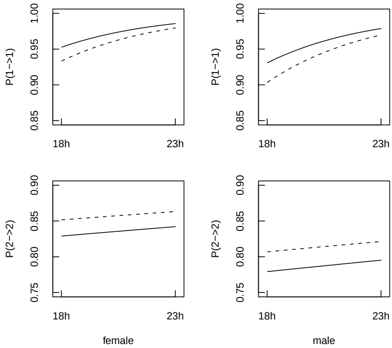

Figure: persistence in behavioural states as function of t
upper plots refer to 'sated' state, lower plots to 'hungry' state
left plots: female lemurs, right plots: male lemurs
solid lines: mass = 59 gr, dashed lines: mass = 33 gr)

Results (part II)

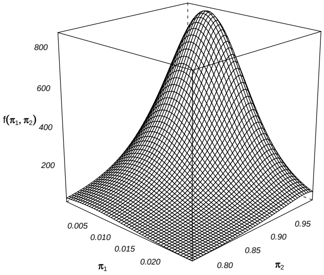

Figure: Fitted joint probability density function of π_1 and π_2.

- notice the scales $\longrightarrow$ much more heterogeneity in π_2, the probability of eating when in the "hungry" state
- doesn't take into account possible correlation between π_i's

Related research

- other types of time series
  - → continuous and/or multivariate data
  - → in particular animal movement paths